

**Chapter 2
Time Value of Money**

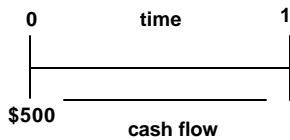
- Future Value
- Present Value
- Annuities
- Effective Annual Rate
- Uneven Cash Flows
- Growing Annuities
- Loan Amortization
- Summary and Conclusions

Basic TVM Concepts

- Interest rate: abbreviated "r" or "k"
 - Compound rate
 - Discount rate
 - Cost of capital
 - Required return
- Present Value – PV – value today
- Future Value – FV – value sometime in the future

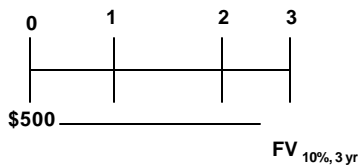
Future Value

- Future value or compounding involves taking today's dollar amount and finding what it will be worth in the future if it earns r percent interest each compounding period.
- If you have \$500 today and it compounds at $i = 10\%$ for one period, you have:

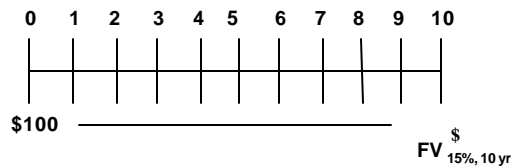


- $FV_1 = PV + \text{Interest} = PV + PV \times i = 500 (1 + 0.10) =$
- For two periods:** $FV_2 = PV (1 + i)(1 + i) = PV (1 + i)^2$
- **For three periods:** $FV_3 = PV (1 + i)^3 = 500 (1.10)^3 =$
- We can generalize this to:**
 $FV_n = PV (1 + i)^n$ or $FV_n = C (1 + i)^n$ or
 $FV_n = PV (FVIF_{i,n})$ using tables

On a time line for 3 years:



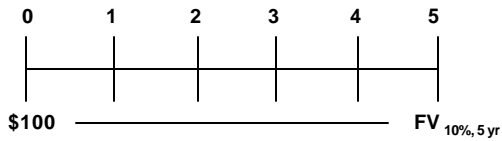
Assume you invest \$100 today, for 10 years at 15% interest, what is the future value in ten years?



$FV_{10} = PV (1 + i)^n =$

$FV_{10} = PV (FVIF_{i,n}) =$

Assume you invest \$100 today, for 5 years at 10% interest, what is the future value in five years?



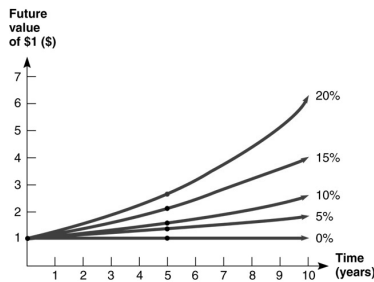
$$FV_{10} = PV(1+i)^n =$$

$$FV_{10} = PV(FVIF_{i,n}) =$$

Future Value Example

- Suppose your great grandfather made a deposit of \$10 at 6% interest 200 years ago. How much would the investment be worth today?
 - $FV =$
- What is the effect of compounding?
 - Simple interest =
 - Compounding added ----- to the value of the investment

Future Value of \$1 at different periods and rates

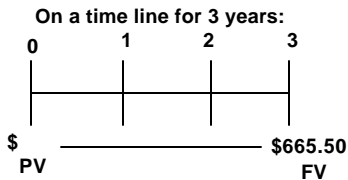


Present Value

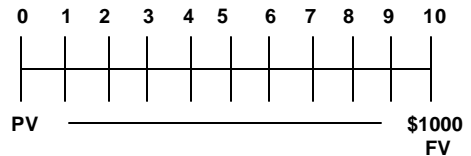
- Notice that $FV_n = PV(1+i)^n$ so rearranging terms we get:
- $PV_n = FV / (1+i)^n$
- Same formula, just solve for PV !!
- Find the PV of 665.50 to be received in 3 years at 10% interest. For three periods:
 $PV = FV / (1+i)^3 = 665.50 / (1.10)^3 = \500.00
- We can generalize this to:
 $PV_n = FV / (1+i)^n$ or $PV_n = C / (1+i)^n$ or
 $PV_n = FV(PVIF_{i,n})$ using tables

Present Value

- Present value or discounting involves taking a future amount and finding what it is worth today, when it is discounted at r percent interest each compounding period.
- If you have \$665.50 in three years and the discount rate is 10%, you have:



Assume you will receive \$1000 in 10 years, your discount rate is 15% interest, what is the present value in of this sum?



$$PV_{10} = FV / (1+i)^n = \text{or}$$

$$PV_{10} = FV(PVIF_{i,n}) =$$

Examples

- You want to begin saving for you daughter's college education and you estimate that she will need \$150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?
 - PV =
- Your parents set up a trust fund for you 10 years ago that is now worth \$19,671.51. If the fund earned 7% per year, how much did your parents invest?
 - PV =

Determining interest rates & periods

- The PV formula, $PV_n = FV / (1 + i)^n$ consists of four factors, present value, future value, interest rate, and number of periods.
- If you know any three of the factors, you can calculate the fourth. To **find the interest rate**, compound rate, discount rate, or rate of return, just solve for i
- $i = (FV / PV)^{1/n} - 1$ or
- Solve for FV / PV and look up this value in the FVIF table under the number of periods row to find the interest rate column or
- Use the financial calculator

Interest Rate Example

- Q. Deposit \$5000 today in an account paying r. If we will get \$10,000 in 10 years, what *rate of return* are we being offered?
- A.

Example continued

- Three ways to find i:
- Solve equation for r:
$$(1 + i)^{10} = 2$$
$$(1 + i) = 2^{1/10} = 1.0717735 - 1$$
$$i =$$
- Solve $FV / PV = 10,000/5,000 = 2$, look up 2 in the FVIF table under the 10 year row, until you find 2, the corresponding interest rate is approximately
- Use the financial calculator.

Interest Rate Examples

- You are looking at an investment that will pay \$1200 in 5 years if you invest \$1000 today. What is the rate of return?
 - i =
- Suppose you are offered an investment that will allow you to double your money in 6 years. You have \$10,000 to invest. What is the implied rate of return?
 - i =

Determining the number of periods

- Once again, you know 3 out of the 4 factors, so solve for i
$$FV = PV (1 + i)^n \Rightarrow (1 + i)^n = FV / PV$$
 then to solve for t, take the log of both sides:
$$n = \ln (FV/PV) / \ln (1 + i)$$
- Or solve for FVIF
$$FVIF = FV / PV$$
 and look up this value in the FVIF table under the proper interest rate column
- Or use your financial calculator

Number of Periods Example

- Q. Deposit \$5,000 today in an account paying 10%. If we will need \$10,000, how long will we have to wait?
- A.

Example continued

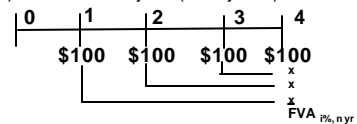
- Three ways to find n:
- Solve equation for n:
 $(1.10)^n = 2$
 $\log(1.10)^n = \log(2)$
 $n = \log(2)/\log(1.10) =$
 $=$
- $FV/PV = 10,000/5,000 = 2$, look up 2 in the FVIF table under the 10% column, this gives approximately --- years for the compounding period.
- Use the financial calculator.

Number of Periods Examples

- You want to purchase a new car and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?
 - $n =$
- Suppose you want to buy some new furniture for your family room. You currently have \$500 and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?
 - $n =$

Valuing an Annuity

- An annuity is a series of equal payments, for a specified number of periods (years or months), with each payment occurring at the end of the period.
- Examples:
- Future Value of an Annuity
- What is the future value of an equal payment (\$100) received each year (for 4 years)?



FVA Continued

- Calculate the FVA of this annuity 4 ways:
- Take FV of each payment individually, for $i = 10\%$:
 $FVA_n = \$100(1+i)^3 + 100(1+i)^2 + 100(1+i)^1 + 100$
 $=$
- Use the formula:

$$FVA_n = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

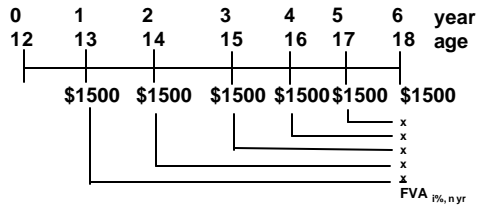
$$FVA = \$100 \left[\frac{(1+.10)^4 - 1}{.10} \right] = 3$$

FVA Continued

- Use table
- $FVA_n = PMT(FVIFA_{i,n}) =$
- Or use the financial calculator:
 $pmt = \$100$, $n = 4$, $i/y = 10$, and $cpt FV$
- Example: Suppose you received an annual gift from your rich grandparents of \$1,500 per year, from your 13th birthday to your 18th birthday, to save for your college tuition. You would invest the money at 10% interest annual at the bank. What is the value of this annuity when you are 18, just after the last payment?

Example continued

The cash flows on a timeline are as follows:



Example continued

Use the formula:

$$FVA_6 = \$1,500 \left[\frac{(1 + .10)^6 - 1}{.10} \right] = \boxed{}$$

Use Table A.4:

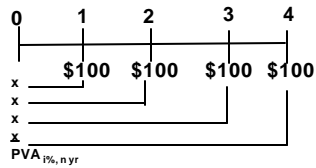
$$FVA_n = PMT(FVIFA_{10\%, 6yr}) =$$

Use the financial calculator:

$$pmt = 1500, n = 6, i/y = 10, \text{cpt } FV$$

Present Value of an Annuity

- What is the present value of an equal payment (\$100) received each year (for 4 years)?



PVA continued

- Calculate the FVA of this annuity 4 ways:
- Take PV of each payment individually, for $i = 10\%$:

$$PVA_4 = \$100/(1+i)^1 + 100/(1+i)^2 + 100/(1+i)^3 + 100/(1+i)^4$$

=

- Use the formula:

$$PVA_n = PMT \left[\frac{1 - 1/(1+i)^n}{i} \right]$$

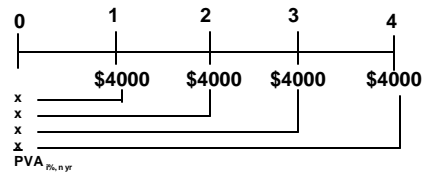
$$PVA_n = \$100 \left[\frac{1 - 1/(1+.10)^4}{.10} \right] =$$

PVA Continued

- Use tables
- $PVA_n = PMT(PVIFA_{i,n}) =$
- Or use the financial calculator:
 $pmt = \$100, n = 4, i/y = 10, \text{ and } \text{cpt } PV$
- Example: Back to the previous example. Suppose you now know that college will cost you \$4,000 per year (cheap college!) for four years. Will the \$11,574 that you saved up in the previous problem be enough to cover the cost of the four years of tuition payments?

Example continued

The cash flows on a timeline are as follows:



Example continued

Use the formula:

Use Tables:

$$PVA_n = PMT(PVIFA_{10\%, 4yr}) =$$

Use the financial calculator:

$$pmt = 4000, n = 4, i/y = 10, \text{cpt PV}$$

Finding the Annuity Payment

- Using the FVA and the PVA annuity formulas we can also calculate the payment, number of periods, and the interest rate.
- Suppose you borrow \$10,000 from your parents to purchase a car. You will make annual payments for 5 years and they require that you pay 6% interest. How much is your annual payment?

Finding the Annuity Payment

- Suppose you want to have \$1 million when you retire in 30 years. You can invest the money at 12% interest. How much do you need to contribute each year?

We can use the financial calculator:

$$FVA = 1 \text{ million}, n = 30, i/y = 12, \text{cpt PMT}$$

Finding the Annuity Period – number of payments

- Suppose you borrow \$2000 at 5% and you are going to make annual payments of \$734.42. How long before you pay off the loan?

$$2000 = 734.42(1 - 1/1.05^n) / .05$$

$$0.136161869 = 1 - 1/1.05^n$$

$$1/1.05^n = 0.863838131$$

$$1.157624287 = 1.05^n$$

$$n = \ln(1.157624287) / \ln(1.05)$$

We can use the financial calculator:

$$PVA = 2,000, PMT = 734.42, i/y = 5, \text{cpt N}$$

Finding the Annuity Interest Rate

- Suppose you invest \$1000 per year for 10 years and you are promised \$18,000 at the end of the 10 years. What is your rate of return?

$$18,000 = 1,000(1 - 1/i^{10}) / i$$

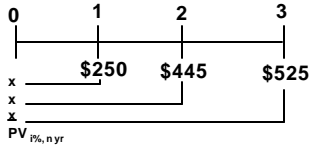
- Two ways to solve for the interest rate:
- Trial & error
- Financial Calculator:

Perpetuities

- Perpetuity is an annuity in which the cash flows continue forever.
- Example: You pay me \$5,000 today and I will repay you \$150 per year for life, for ever & ever, even to your heirs after you die. I guarantee it forever. Will you take my offer?

Present Value of an Uneven Series

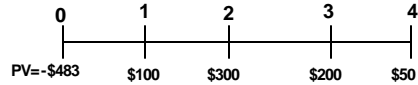
An uneven series means that the yearly cash flows are different each year, unlike an annuity. An example is shown below.



To solve this, you must present value each cash flow separately. For $r = 10\%$, and using tables, you have:

Solving for an Uneven Series Interest Rate

Assume you have the following cash flows and you want to know the interest rate. Or what is your rate of return on this investment?



There are two ways to solve this, trial and error or financial calculator. For trial and error, choose an interest rate, solve for the PV of each cash flow and compare the total PV to the actual PV. If it is not the same, then select a new interest rate and try again.

If your answer was too high, select a higher interest rate (more discounting), if it was too low, select lower interest rate (less discounting).

Uneven series interest rate continued

Start with 14%, and put in the form of a table:

Year	Cash Flow	PV at 14%	PV
1	100	.8772	87.72
2	300	.7695	230.85
3	200	.6750	135.00
4	50	.5921	29.61
		Total =	29.61

Compare this answer with the original PV, it's the same, so we are done. Otherwise we would adjust the interest rate up or down and try it again. So our rate of return on this investment is -----

EAR

The Effective Annual Interest Rate (EAR) is the annual rate that includes the effect of compounding multiple times during the year

To convert a quoted rate to an EAR:

$$EAR = [1 + (\text{quoted rate}/m)]^m - 1 \quad (\text{quoted rate} = i_{nom})$$

where $m = \text{no. of compounding periods}$

$$EAR = \left(1 + \frac{i_{nom}}{m}\right)^m - 1$$

For example: The bank quotes you a rate of 12%, compounded quarterly. A friend quotes you a nominal rate of 11.9% compounded daily. Which rate is better?

$$EAR_{\text{bank}} =$$

$$EAR_{\text{friend}} =$$

Continuous Compounding

- To find the future value of an investment compounded continuously over many periods:

$$FV = PMT_0 \times e^{in}$$

Where

PMT_0 = cash flow at time 0

i = stated annual interest rate

n = number of periods

e is a constant, $\cong 2.718$

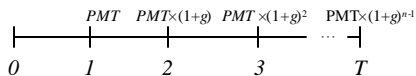
(e^x is a key on your calculator)

Effective Annual Rates and Compounding

Compounding period	Number of times compounded	Effective annual rate
• Year	1	10.00000%
• Quarter	4	10.38129
• Month	12	10.47131
• Week	52	10.50648
• Day	365	10.51558
• Hour	8,760	10.51703
• Minute	525,600	10.51709
• Continuous	continuous	10.51709

Growing Annuity

A growing stream of cash flows with a fixed maturity.



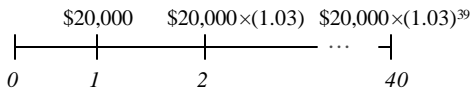
$$PV = \frac{PMT}{(1+i)} + \frac{PMT \cdot (1+g)}{(1+i)^2} + \dots + \frac{PMT \cdot (1+g)^{n-1}}{(1+i)^n}$$

The formula for the present value of a growing annuity:

$$PV = \frac{PMT}{i-g} \left[1 - \frac{1+g}{1+i} \right]^n$$

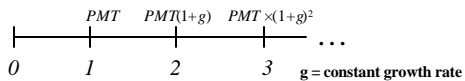
Growing Annuity

A defined-benefit retirement plan offers to pay \$20,000 per year for 40 years and increase the annual payment by 3-percent each year. What is the present value at retirement if the discount rate is 10-percent?



Growing Perpetuity

A growing stream of cash flows that lasts forever.



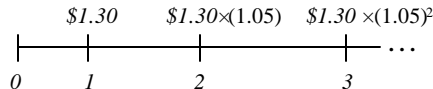
$$PV = \frac{PMT}{(1+i)} + \frac{PMT \cdot (1+g)}{(1+i)^2} + \frac{PMT \cdot (1+g)^2}{(1+i)^3} + \dots$$

The formula for the present value of a growing perpetuity is:

$$PV = \frac{PMT}{i-g}$$

Growing Perpetuity: Example

The expected dividend next year is \$1.30 and dividends are expected to grow at 5% forever. If the discount rate is 10%, what is the value of this promised dividend stream?



Amortized Loans

- Requires the borrower to pay the interest plus part of the principal each payment.
- The process of paying off the principal and interest over time is called amortizing the loan.
- To see the interest and principal paid each period, you would prepare a loan amortization schedule. This is normally provided for home mortgages and similar loans.
- To set up a loan amortization schedule you start by calculating the payment.
- For example: Assume you borrow \$66,242 to be repaid over the next four years at 8% interest.

Your payment would be:

$PVA = PMT (PVFA_{i,n})$ and solve for the payment:

Loan Amortization continued

Set up the table as follows:

Year	Payment	Interest	Principal	Balance
1				
2				
3				
4				